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Nonlinear Tracking Control of Non-minimum Phase System Using Input-Output Linearization and Vanishing Perturbation Theory

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Abstract. In an attempt to deal with the problem of tracking control for the nonlinear system, a non minimum phase is considered. Indeed, the main idea here is to neglect a part of the system dynamics so as to make the approximate system input-state feedback linearizable. The neglected part is then considered as a perturbation. Also, a linear controller is designed to control the approximate system. Stability is analyzed using the vanishing perturbation theory. The performance of the proposed approach is evaluated in an illustrative inverted cart-pendulum example.

Keywords: Tracking Control, Non-minimum Phase System, Observability Normal Form, Vanishing Perturbation.

1 Introduction

The tracking control for nonlinear non minimum systems is a challenging problem in the control theory [4][8][14]. The standard input-output linearization [5]-[11]-[12] leads to an unstable closed loop system due to the presence of unstable zero dynamics. Hence, various ideas related to the possibility of using input-output linearization have been explored in the literature dealing with the nonlinear non-minimum phase system. In [3], a nonlinear state feedback and a coordinate transformation are used to make the system as close as possible to a linear one. In

[2], the system input output feedback is first linearized. Then, the zero dynamics is factorized into stable and unstable parts. The unstable part is approximately linear and independent of the coordinates of the stable part. [6] Proposed a numerical approach applying multivariable legendre polynomials to achieve an exact algebraic expression for the exact linearizing feedback. On the other hand, a cascade control scheme has been considered that combines the input-output feedback linearization and the backstepping approach [5].
In this paper, we address the problem of tracking control of a single input single output of non minimum phase nonlinear systems. The idea here is to approximate the given system into input state feedback linearizable system. The system is feedback linearized by neglecting a part of the system dynamics, with the neglected part being considered as a perturbation. Stability analysis is also provided based on the vanishing perturbation theory [13].

The present paper is organized as follows: in Section 2 some mathematical preliminaries are presented. The cascade control law design and the stability analysis are given in Section 3. Section 4 gives the inverted cart-pendulum to illustrate the effectiveness of the proposed approach. Finally, some concluding remarks are provided in Section 5.

2 Preliminaries and Problem Statement

In this paper, we consider a nonlinear Single-Input Single-Output (SISO) system of the form:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \quad x(0) = x_0 \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in \Re^n$ is the n-dimensional state variables, $u \in \Re$ is a scalar manipulate input and $y \in \Re$ is a scalar output. $f(.)$, $g(.)$ and $h(.)$ are smooth functions describing the system dynamics.

2.1 Input-Output Linearization

Consider the output $y = h(x)$ for system (1). The nonlinear system (1) has relative degree r at the point x_0 if:

$$\begin{aligned} & L_f^k h(x) = 0 \quad \forall x \neq x_0 \text{ and } \forall k \leq r-1 \\ & L_g L_f^{(k-1)} h(x) \neq 0 \end{aligned} \tag{2}$$

So, the relative degree r is the number of times we have to differentiate the output y with respect to time before the input appears [11].

If $y \le n$, then system (1) can be feedback linearized into Byrnes-Isidori normal form [10]:

$$\xi = \begin{bmatrix} h(x) & \dots & L_f^r h(x) & \eta_1 & \cdots & \eta_{n-r} \end{bmatrix}^T \tag{3}$$

The resulting system with the transformed variables (1) can be written as:

$$\begin{aligned} &\dot{\xi}_i = \xi_{i+1} \quad i = 1,\dots,r-1 \\ &\dot{\xi}_r = v = L_f^r h(x) + L_g L_f^{r-1} h(x) u \\ &\dot{\eta} = q(\xi,\eta) \\ &y = \xi_1 \end{aligned} \tag{4}$$

where v is the new control law.
Thus, the control law can be written as:

$$u(x) = \frac{v - L_f^r h(x)}{L_g L_f^{r-1} h(x)} \tag{5}$$

2.2 Vanishing Perturbation Theory

In this section, we consider the nonlinear system (1) which is written in the autonomous form for the following perturbed system:

$$\dot{x} = f(x) + \Theta(x), \quad x(0) = x_0 \tag{6}$$

where $f(x)$ represents the nominal dynamics, with $f(0) = 0$, and $\Theta(x)$ represents the perturbed dynamics. f and Θ are Lipchitz in x. The vanishing perturbation theory is based on the assumption that the perturbation tends towards zero in the origin $\Theta(0) = 0$.

Thus, if the nominal system is exponentially stable and the parameter λ is smaller than a predetermined limit, then the perturbed system is also exponentially stable.

Theorem 1 [13]: *Let $(x = 0)$ is the equilibrium point of the nominal system $\dot{x} = f(x)$ which is exponentially stable and $V(x)$ is a Lyapunov function of the nominal system that satisfies the following conditions:*

$$\textbf{\textit{i.}} \quad \frac{\partial V(x)}{\partial x} f(x) \le -c_1 \|x\|^2 \tag{7}$$

$$\textbf{\textit{ii.}} \quad \left\| \frac{\partial V(x)}{\partial x} \right\| \le c_2 \|x\| \tag{8}$$

with c_1 and c_2 are two real positive constants.

Let $\exists \lambda > 0 : \|\Theta(x)\| \le \lambda \|x\|$, so, if $\lambda < \frac{c_1}{c_2}$, the origin $(x = 0)$ is an exponentially stable equilibrium point of the perturbed system (6).

3 Main Results

In this paper, an approach to the tracking control problem of the nonlinear non minimum phase system is proposed based on the vanishing perturbation theory. First, the nonlinear non-minimum phase system is transformed into its observability normal form. Next, it is in turn approximated as a chain of integrators by neglecting a part of the dynamics. Finally, it is controlled via a linearizing feedback.

3.1 Controller Design

For dealing with the SISO system in (1), the following assumptions are first made.

Assumption 1:

$$\forall x \in \Re^n \quad span\left\{\begin{bmatrix} \frac{\partial h}{\partial x} & \frac{\partial L_f h}{\partial x} & \frac{\partial L_f^2 h}{\partial x} & \cdots & \frac{\partial L_f^{n-1} h}{\partial x} \end{bmatrix}\right\} = n \tag{9}$$

where $\frac{\partial h}{\partial x}, \frac{\partial L_f h}{\partial x}, \frac{\partial L_f^2 h}{\partial x}, \cdots$ *and* $\frac{\partial L_f^{n-1} h}{\partial x}$ *are linearly independent[10].*

This assumption implies that the linearized system is observable around all operating points [15].

Assumption 2: *Let the following state transformation:*

$$\xi = \Phi(x) = \begin{bmatrix} h(x) & L_f h(x) & L_f^2 h(x) & \dots & L_f^{n-1} h(x) \end{bmatrix}^T \tag{10}$$

$\Phi(x)$ *is a diffeomorphism, i.e., the inverse transformation* $x = \Phi^{-1}(\xi)$ *exists and is unique for all* $x \in \Re^n$.

We start by defining the following error states:

$$e_i = \xi_i - y_{ref}^{(i-1)} \quad \forall i \in \{1, 2, \dots, n\} \tag{11}$$

where y_{ref} is a reference trajectory

using the state transformation (10) and the linearized feedback control (5), the system (1) is written as follows:

$$\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 \\
&\vdots \\
\dot{e}_r &= e_{r+1} + L_g L_f^{r-1} h\left(\Phi^{-1}(\xi)\right) \\
\dot{e}_{r+1} &= e_{r+2} + L_g L_f^{r} h\left(\Phi^{-1}(\xi)\right) \\
&\vdots \\
\dot{e}_n &= L_f^n h\left(\Phi^{-1}(\xi)\right) + L_g L_f^{n-1} h\left(\Phi^{-1}(\xi)\right) u - y_{ref}^n, \quad e(0) = e_0
\end{aligned} \tag{12}$$

with $L_g L_f^i h\left(\Phi^{-1}(\xi)\right) = 0$, $\forall\ 0 < i < r-1$ and $L_g L_f^i h\left(\Phi^{-1}(\xi)\right) \neq 0$, $\forall\, r-1 \le i \le n-1$.
The main idea of the proposed approach is to assume that the terms $L_g L_f^i h\left(\Phi^{-1}(\xi)\right) \neq 0$, $\forall\, r-1 \le i \le n-2$ are towards zero and are, then, neglected.
Thus, the use of this assumption in (12) yielding the following approximate system:

$$\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 \\
&\vdots \\
\dot{e}_r &= e_{r+1} \\
\dot{e}_{r+1} &= e_{r+2} \\
&\vdots \\
\dot{e}_n &= L_f^n h\left(\Phi^{-1}(\xi)\right) + L_g L_f^{n-1} h\left(\Phi^{-1}(\xi)\right) u - y_d^{(n)}, \quad e(0) = e_0
\end{aligned} \tag{13}$$

The resulting system (13) consists of a chain of n integrators, and then the following linearizing control can be applied:

$$u = \frac{v - L_f^n h\left(\Phi^{-1}(\xi)\right)}{L_g L_f^{n-1} h\left(\Phi^{-1}(\xi)\right)} \tag{14}$$

Then, system (13) can be rewritten as follows:

$$\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 \\
&\vdots \\
&\vdots \\
\dot{e}_n &= v - y_{ref}^{(n)}, \ e(0) = e_0
\end{aligned} \tag{15}$$

So, it may be written as:

$$\dot{e} = Ae + Bv, \ e(0) = e_0 \tag{16}$$

with $A = \begin{bmatrix} 0 & 1 & 0 & . & 0 \\ 0 & 0 & 1 & . & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & . & 1 \\ 0 & 0 & . & . & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$.

The resulting system (16) is composed of a linear one and corresponds to an approximate input-state input of system (1). Consequently, the following feedback:

$$v = y_{ref}^{(r)} + \sum_{i=0}^{r-1} k_{i+1}\left(y_{ref}^{i} - y^{i}\right) \tag{17}$$

can be applied to the approximate system (16). We obtain, finally, the following linear system in closed loop:

$$\dot{e} = A_{CL}\, e, \quad e(0) = 0 \tag{18}$$

with: $$A_{CL} = \begin{bmatrix} 0 & 1 & 0 & . & 0 \\ 0 & 0 & 1 & . & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & . & 1 \\ -k_1 & -k_2 & . & . & -k_n \end{bmatrix}$$

and $k_i > 0, \forall i \in \{1, 2, \ldots, n\}$ are the coefficients of a Hurwitz polynomial [13].

3.2 Stability Analysis

In this section, we use the theory of vanishing perturbation to analyze the stability of the closed loop system. The application of the linear feedback (17) to the transformed system (14) leads to:

$$\dot{e} = A_{CL} e + \Theta\left(\Phi^{-1}(\xi)\right), \quad e(0) = e_0 \tag{19}$$

with:

$$\Theta\left(\Phi^{-1}(\xi)\right) = -\begin{bmatrix} 0 \\ \vdots \\ L_g L_f^{r-1} h\left(\Phi^{-1}(\xi)\right) \\ \vdots \\ L_g L_f^{n-2} h\left(\Phi^{-1}(\xi)\right) \\ 0 \end{bmatrix} \frac{\left(Ke(t) + L_f^n h\left(\Phi^{-1}(\xi)\right)\right)}{L_g L_f^{n-1} h\left(\Phi^{-1}(\xi)\right)} \tag{20}$$

Then, replacing (ξ, u) by their values at the equilibrium $(\xi, u) = (0, 0)$ in (12) gives $L_f^{n-1} h\left(\Phi^{-1}(\xi = 0)\right) = 0$. Replacing it in (20) yields $\Theta(0) = 0$. Therefore, the perturbation is indeed vanishing, and the theory of vanishing perturbations can be used. The following theorem gives sufficient conditions for the exponential stability of system (19).

Theorem 2 [7]: *Consider system (1) and assume that the following conditions are verified:*

- $\forall x \in \Re^n,\ span\left\{\left[\frac{\partial h(x)}{\partial x} \quad \frac{\partial L_f h(x)}{\partial x} \quad \frac{\partial L_f^2 h(x)}{\partial x} \quad \cdots \quad \frac{\partial L_f^{n-1} h(x)}{\partial x}\right]\right\} = n,$
- $\exists \delta_1 > 0,\ \left|L_f^n h\left(\Phi^{-1}(\xi)\right)\right| \le \delta_1 \|\xi\|$,

- $\exists\delta_2 > 0,\ \left|L_g L_f^{n-1} h\left(\Phi^{-1}(\xi)\right)\right| \geq \delta_2$
- *The gains* $k_i, (i = 0,\ldots,n)$ *are the solutions of Hurwitz polynomial.*
- *Let us consider the function* $\Theta^*(\xi)$ *defined by:*

$$\Theta^*(\xi) = \left[0 \ \ldots \ 0 \ \ L_g L_f^{r-1} h\left(\Phi^{-1}(\xi)\right) \ \ldots \ L_g L_f^{n-2} h\left(\Phi^{-1}(\xi)\right) \ \ 0\right]^T \quad (21)$$

If

$$\left\|\Theta^*(\xi)\right\| < \frac{\lambda_2}{2\alpha_{max}(P)\left(\lambda_1 + \|K\|\right)} \quad (22)$$

where P *is the solution of Lyapunov equation* $P A_{CL} + A_{CL}^T P = -I$

Thus, the control law (14)-(17) stabilizes system (12) exponentially.

Proof: Consider Lyapunov function $V = e^T P e$ for system (18), where P is a positive symmetric matrix that satisfies Lyapunov equation $PA + A^T P = -I$. Then,

$$\frac{\partial V}{\partial e} A_{CL} e = e^T PAe + e^T A^T Pe = -\|e\|^2 \quad (23)$$

and:

$$\begin{aligned} \left\|\frac{\partial V}{\partial e}\right\| &= \left\|e^T P + P e\right\| \leq 2\,\|e\|\ \|P\| \\ &\leq 2\,\alpha_{max}\|e\| \end{aligned} \quad (24)$$

Consider the same Lyapunov function for the perturbed system (19), given by:

$$\dot{V} = \frac{\partial V}{\partial e} A_{CL} e + \frac{\partial V}{\partial e}\Theta(\xi) \leq -\|e\|^2 + 2\alpha_{\max}(P)\|e\|\ \|\Theta(\xi)\| \quad (25)$$

Noting that $\left|L_f^n h\left(\Phi^{-1}(\xi)\right)\right| \leq \lambda_1\|\xi\|$ and $\left|L_g L_f^{n-1} h\left(\Phi^{-1}(\xi)\right)\right| \geq \lambda_2 \geq 0$ so by using the expression $\Theta(\xi)$ (20), yields:

$$\|\Theta(\xi)\| \leq \frac{\lambda_1 + \|K\|}{\lambda_2}\ \|\Theta^*(\xi)\|\ \|\xi\| \quad (26)$$

Replacing (26) in expression (25) gives:

$$\dot{V} \leq -\|e\|^2\left(1 - 2\alpha_{\max}(P)\frac{\lambda_1 + \|K\|}{\lambda_2}\ \|\Theta^*(\xi)\|\right) \quad (27)$$

Thus, if (22) is verified, the term in the parenthesis is positive and the feedback law (14)-(17) stabilizes (12) exponentially. The following corollaries are related to Theorem 2.

Corollary 1: *For a given vector gain* K *the closed loop system exponentially stabilizes (19) for all:* $\|\Theta^*(\xi)\| < \dfrac{\lambda_2}{2\alpha_{\max}(P)\left(\lambda_1 + \|K\|\right)}$

The corollary above declares that, for any given vector gain K there is a range different from the non-null values of $\Theta^*(\xi)$ for which system (12) can be stabilized. However, the converse is not true, i.e., that is to say, given a perturbation $\Theta^*(\xi)$, it is not always possible to find a gain vector K that stabilizes the system.

Corollary 2: *Let* λ_1 *and* λ_2 *and* $\lambda^* = \max_K \frac{\lambda_2}{2\lambda_{\max}(P)(\lambda_1 + \|K\|)}$, $eig(A_{CL}) < 0$. *For* $\Theta^*(\xi) < \lambda^*$, *there exists a vector gain* K *that stabilizes system (19) exponentially.*

4 Illustrative Example

4.1 Description of the Inverted Cart- Pendulum System

Consider the familiar inverted cart-pendulum system [1], depicted in Fig. 1. The cart must be moved using the force u so that the pendulum remains in the upright position as the cart tracks any desired trajectory from a class of admissible trajectories.

Let the mass of the cart be M, the mass of the pendulum be m, the length of the stick be L, and the acceleration of the gravity be g. The mass of the stick is small compared with the mass m and will be neglected by choosing the pendulum angle θ and the cart position y_c as the generalized position coordinates for the system. The effect of friction is also neglected.

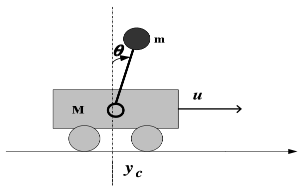

Fig. 1. A Schematic representation of the Inverted Cart-Pendulum System.

The inverted cart-pendulum equations are:

$$\begin{aligned} \ddot{y}_c &= \frac{u + m\left(L\dot{\theta}^2 - g\cos\theta\right)\sin\theta}{M + m\left(\sin(\theta)\right)^2} \\ \ddot{\theta} &= \frac{1}{L}\left(g\sin(\theta) - \ddot{y}\cos(\theta)\right) \end{aligned} \tag{28}$$

Let $x = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}^T = \begin{pmatrix} \theta & \dot{\theta} & y_c & \dot{y}_c \end{pmatrix}^T$ and $y = x_3$.Then, we obtain the following state space equation

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{L}\left(g\sin\left(x_1\right) - \dot{x}_4\cos\left(x_1\right)\right) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{u + m\left(Lx_2^2 + g\cos\left(x_1\right)\right)\sin\left(x_1\right)}{M + m\sin\left(x_1\right)^2} \\
y &= x_3
\end{aligned} \tag{29}$$

The initial conditions are $y_c(0) = \dot{y}_c(0) = \dot{\theta}(0) = 0$ and $\theta(0) = -\pi$, the downward position for the pendulum. The system parameters are given in Table 1.

Table 1. Numerical Parameters of the Inverted Cart-Pendulum System

Parameters	Signification	Numerical value
M	Mass of the cart	$0.455\ kg$
m	Mass of the rod	$0.21\ kg$
l	Length of the rod	$0.355m$
g	Gravitational acceleration	$9.81\ m/s^2$

4.2 Control Law

The objective here is to control the cart displacement y_c along a reference trajectory and stabilize the pendulum angle θ to the upright position. Thus, the proposed approach to the system (2) is applied. Again, the Lie derivatives are computed up to the order of the system

$h = x_3$, $L_f h = x_4$, $L_f^2 h = T_1$, $L_f^3 h = T_2$, $L_f^4 h = T_3$, $L_g h = 0$, $L_g L_f h = 1/(\mathrm{M} + m\sin\left(x\right)^2)$, $L_g L_f^2 h = T_4$, $L_f^3 h = T_5$. The length terms T_i, $i = 1,\ldots,5$ are included in the appendix. Since h, $L_f h$, $L_f^2 h$, $L_f^3 h$ are independent, the system can be written in the observability normal form (14) given by:

$$\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 + L_g L_f h\left(x\right)u \\
\dot{e}_3 &= e_4 + L_g L_f^2 h\left(x\right)u \\
\dot{e}_4 &= L_f^4 h\left(x\right) + L_g L_f^3 h\left(x\right)u - y_{cref}^4, \quad e\left(0\right) = e_0
\end{aligned} \tag{30}$$

and the approximate system, after neglecting the internal dynamics, is given by:

$$\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 \\
\dot{e}_3 &= e_4 \\
\dot{e}_4 &= v - y_{cref}^{(4)}, \quad e\left(0\right) = e_0
\end{aligned} \tag{31}$$

Finally, the approximate system is linearized using the following control law:

$$u = \frac{v - L_f^4 h(x)}{L_g L_f^3 h(x)} \tag{32}$$

where v is a linear feedback control:

$$v = y_{cref}^{(4)} - k_1\left(y_{cref} - y_c\right) - k_2\left(\dot{y}_{cref} - \dot{y}_c\right) - k_3\left(\ddot{y}_{cref} - \ddot{y}_c\right) - k_4\left(y_{cref}^3 - y_c^3\right) \tag{33}$$

with $y_{cref}(t)$ is the desired trajectory to follow and the gains k_1, k_2, k_3 and k_4 are the solutions of Hurwitz polynomial.

4.3 Simulations Results

In simulation, the reference trajectory used in the proposed approach is $y_{cref} = 1.5(\sin(\mathrm{t}) + \sin(0.5\,\mathrm{t}))$ and the gains values are
$k_1 = -2.71,\ k_2 = -2.72, k_3 = -2.73,\ k_4 = -2.74$.

The simulation result of the tracking control is shown in Figure 2. This figure presents a perfect agreement between the two trajectories. Figure 3 shows the evolution of the pendulum angle; indeed, it is a small variation around zero. The evolution of the stabilizing control law is shown in Figure 4. The dynamics of this control signal is quite satisfactory.

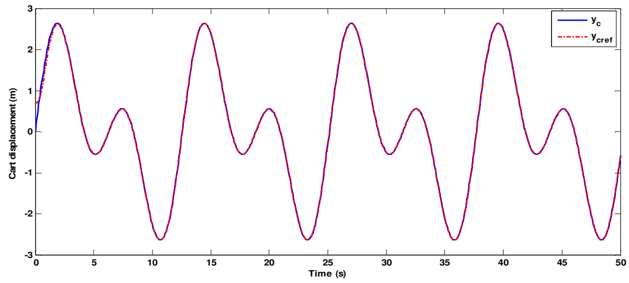

Fig. 2. Evolution of the cart displacement y_c and the reference trajectory y_{cref}

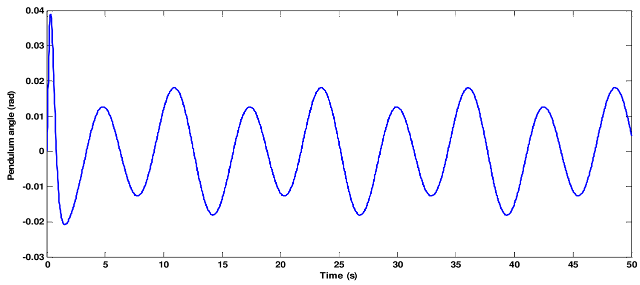

Fig. 3. Evolution of the angle pendulum θ

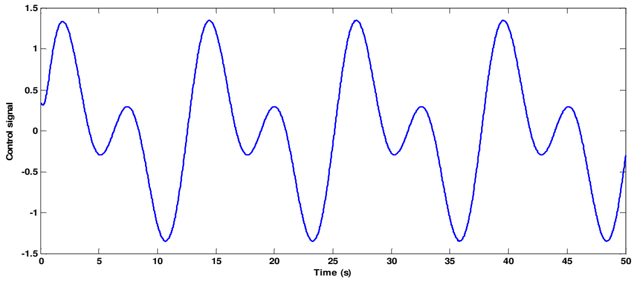

Fig. 4. Evolution of the control signal

5 Conclusion

In this paper, based on an approximate input-state feedback linearization technique, we present a new control scheme to solve the problem of tracking control for the nonlinear non minimum phase system. The nonlinear system is first transformed into its observability normal form. The latter is in turn approximated as a chain of integrators, neglecting part of the dynamics, and is finally controlled via a linearizing feedback.

The neglected part is considered as a perturbation, which is vanishing at the origin. Thus, the stability analysis is provided based on the vanishing perturbation theory. The efficiency and the validity of the proposed approach are illustrated through an example of inverted cart-pendulum.

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Appendix

$$T1 = m(lx_2^2 - g\cos(x_1))\sin(x_1) / (l(M + m\sin(x_1)^2))$$

$$T_2 = -(-3x_2 gM + 4x_2 g\cos(x_1)^2 M - 3x_2 gm + 3x_2 gm\cos(x_1)^2 + 3lm\cos(x_1)x_2^3 - \cos(x_1)lx_2^3 M$$
$$+ \sin(x1)lx_2^2 M - \sin(x_1)g\cos(x_1)M)m / (l(M + m\sin(x_1)^2)^2)$$

$$T_3 = -(-6x_2^3 gM - 12x_2^2 Mg\sin(x_1)^2 M - 6x_2 gm + 6x_2 gm\cos(x_1)^2 + 2x_2 gm^2\cos(x_1)^2$$
$$- \cos(x_1)l^2 x_2^3 M + 6lm\cos(x_1)x_2^3 - \cos(x_1)lx_2^3 M + \sin(x1)lx_2^3 M + gMsin(x_1)^2\cos(x_1)$$
$$- 2\sin(x_1)g\cos(x_1)M)m / (l(M + m\sin(x_1)^2)^4)$$

$$T_4 = l(M + 2mx_2\sin(x_1)) - 3m\cos(x_1)^3 lx_2^3 - 2x_2\sin(x_1)\cos(x_1) / (l(M + m\sin(x_1)^2)^2)$$

$$T_5 = l(2x_2\sin(x_1)g\cos(x_1)M) - m\sin(x_1)\cos(x_1)^3 lx_2^3 - 3m\cos(x_1)^3 lx_2^4$$
$$- 2x_2^2\,\mathrm{gM}\sin(x_1)\cos(x_1) / (l(M + m\sin(x_1)^2)^4)$$